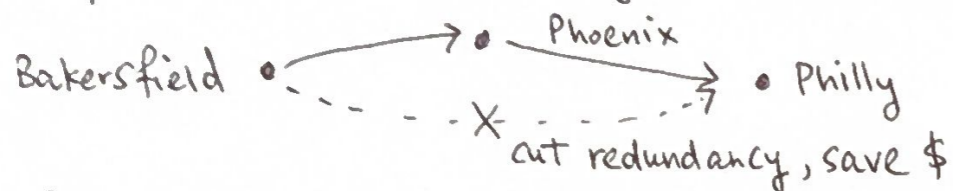


19 Minimum spanning trees (MST)

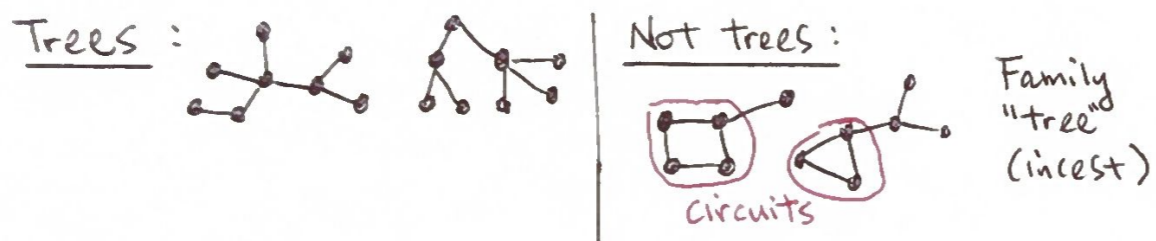
Why airlines offer transfer flights instead of direct flights?



Graph with no redundancy is a "tree".

Defn A circuit in a graph is a walk from a vertex to itself that reuses no edge.

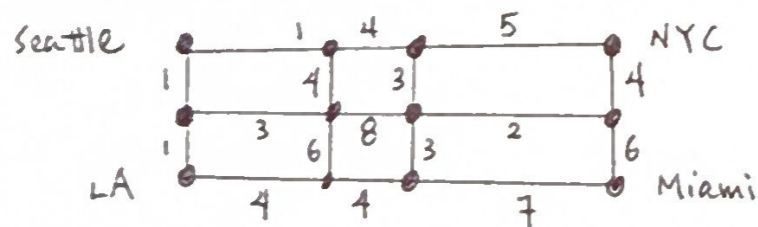
A tree is a connected graph with no circuits.



To get bigger tree, add 1 new vertex via 1 new edge to the existing tree. So:

Euler's formula for trees: $G=(V,E)$ is a tree iff $|V| = |E| + 1$ to account for initial vertex

Problem Airline has routes (edges) to cities (vertices) with route costs below:



Such graph with edge weights is a weighted graph.

Problem To save \$, airline cuts all redundant routes.

So we want tree subgraph spanning all vertices while minimizing edge weights.

This is a minimal spanning tree (MST) of the graph.

Kruskal's algorithm: To get MST, add lowest cost edges without forming circuits, until you can't anymore.

Soln to problem

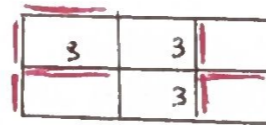
First add lowest cost edges (1's)



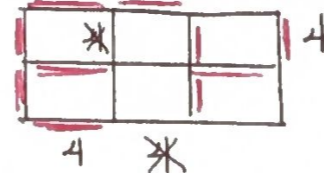
Add next lowest cost edges (2's)



Add 3's:



Add 4's:



lower 4's or else we form a circuit. can't add both

Add 5's: can't! (forms circuit!)



Add 6's:



Every vertex is in our tree network, so this is a MST by Kruskal's algorithm.

MST applications: Utility lines, distribution networks.

In real world, networks have some redundancies to improve reliability/network resilience.